Dispersion of passive tracers in the direct enstrophy cascade: Experimental observations

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The paper reports a statistical analysis of the separation of pairs in the enstrophy cascade, in a two-dimensional flow. The flow is produced experimentally, using electromagnetic forcing. Two regimes of separation are found. At early times (i.e., within two integral times) the separation process is exponential, following Lin’s law [J. T. Lin, J. Atmos. Sci. 29, 394 (1972)]. In this range of time, the probability density functions (PDFs) of separations are self-similar in time, developing stretched exponential tails, and one may define a Lagrangian correlation time. Above two integral times, an algebraic regime takes place, with exponential tails for the PDFs of the separations, and self-similar Lagrangian correlation functions. The present work thus confirms the existence of two regimes, by analyzing a number of statistical quantities including the Lagrangian PDF and the temporal Lagrangian correlation functions which so far, were left undetermined for the Batchelor regime. © 2003 American Institute of Physics. [DOI: 10.1063/1.1585030]

I. INTRODUCTION

The theoretical analysis of tracer fields is simpler than for the velocity, since the equation for the dispersion of a tracer is linear with the concentration. Let us call $\Theta = \langle \Theta \rangle + \theta$ the concentration field. One has

$$\frac{\partial \Theta(x,t)}{\partial t} + u(x,t) \cdot \nabla \Theta(x,t) = D \nabla^2 \Theta(x,t) + f(x,t),$$

where $\theta$ is the concentration fluctuation, $\langle \Theta \rangle$ the mean concentration of tracer field, and $f(x,t)$ is the forcing.

At the moment, much is known on the Eulerian properties of the tracer field, for the case of the Batchelor regime, i.e., when the velocity field is smooth (the other case, i.e., a rough velocity field, has not been solved yet in the general case). Recently, Chertkov et al. and Falkovich et al. were able to “solve” the statistical problem, i.e., to analytically find expressions for the main statistical moments of the concentration field, i.e., the structure functions at any order. At second order, they recover the spectrum, defined by $\langle \theta^2 \rangle = \int_0^\infty dk E(k)$, which has the form

$$E(k) \sim \frac{\chi}{\gamma} k^{-1},$$

where $\chi = 2 \kappa (\nabla \theta)^2$ is the rate at which scalar variance is destroyed, and $\gamma$ is the mean rate of strain. This result, known as the Batchelor spectrum was derived by Batchelor and can be deduced from dimensional arguments à la Kolmogorov. The calculation of Chertkov et al. led to determine the probability density functions (called later PDFs) of both scalar fluctuations and increments. These theoretical findings could be confirmed experimentally by Jullien et al., and more recently by Groisman and Steinberg. We now are in a pleasant situation where theory and experiment agree, and one may consider the problem is somehow “solved.” These experiments considered Eulerian aspects, and at the moment, much less is known about Lagrangian quantities. It would nonetheless be instructive to investigate the Lagrangian aspects of the problem for the following reasons: in two dimensions, the study of Lagrangian properties may allow to reveal logarithmic corrections of the enstrophy cascade, when the Batchelor regime takes place in the same range of scales as the enstrophy cascade (which happens in the experiment reported here). Second, the Lagrangian properties of the Batchelor regime are so far poorly known from the experimental viewpoint (for instance, we do not know the PDFs of the pair separation). This has motivated the present work.

The classical argument leading to the separation law for a pair of particles released in a smooth flow is the following: the velocity increments scale as $v \sim ar$. The root mean square displacement for a pair distribution then follows

$$\frac{dR^2}{dt} \sim R^2,$$

known as Lin law. In the particular case of an enstrophy cascade, the parameter $a$ is $a = \beta^{1/3}$, where $\beta$ is the enstrophy rate transfer.

By integrating, the root mean square displacement evolves as

$$R^2 \sim R_0^2 \exp \left[ c \frac{t}{\tau_*} \right],$$

where $R_0$ is the initial separation, $c$ is a nondimensional constant and $\tau_*$ the characteristic dispersion time, defined by the enstrophy cascade rate, i.e., $\tau_* = \tau = \beta^{-1/3}$.

However, if the flow at hand is produced by an enstrophy cascade, the enstrophy cascade spectrum $E(k)$...
leads to a logarithmic divergence of the total enstrophy when the viscosity tends to zero. Consequently, Kraichnan\cite{Kraichnan} introduced a logarithmic correction to the spectrum coming from the nonlocal characteristic of nonlinear interactions: strain acting on small scales comes mainly from larger scales. These arguments lead to the following corrected spectrum:

$$E(k) = C' \beta^{2/3} k^{-3} \left( \log(k/k_1) \right)$$

with \(k \gg k_1\). \hspace{1cm} (4)

This correction yields a modified expression for the mean square separation,\footnote{where Lin's law applies} leading to a more pronounced effect on the separation process:

$$R^2 = R_0^2 \exp \left( 2 c^n \beta^{1/3} t^{3/2} \right),$$

where \(c^n\) is a numerical constant. \hspace{1cm} (5)

At the present time, there are several experimental and numerical investigations of the Lagrangian properties of Batchelor regime, related or not to an enstrophy cascade. On the experimental side, the EOLE campaign consisted in following 480 constant-volume balloons released in the Southern Hemisphere at about 200-mb level. Morel and Larcheveque\cite{Morel} have observed an exponential growth of the mean square separation, in accordance with Lin law. On the numerical side, Babiano et al.\cite{Babiano} observed that the exponential law takes place at early times in a range of scales shorter than the integral scales. Kowalski and Peskin,\cite{Kowalski} in the case of two-dimensional, homogeneous, decaying turbulence, showed that an exponential behavior is observed at early times, and that a power law regime \((\sigma^2 - t^n\) with \(n\) approximately 2.3) is observed for larger times. In the present paper, I extend these observations. In particular, I confirm the existence of two regimes, one at early times, exponential (where Lin’s law applies), and the other algebraic (at larger times), and determine its characteristics. It is appropriate to stress at this stage that, in the present paper, the Lagrangian trajectories are inferred from the experimentally measured velocity fields. Several issues (well separated characteristic lengths scale and weak intermittency) have been raised concerning this procedure by Artale et al.\cite{Artale} and Boffetta et al.,\cite{Boffetta} who gave an original approach by introducing the finite size Lyapunov exponent (FSLE). In order to strength the present analysis, the classical method as well as the FSLE’s one are both used for analyzing the dispersion problem, for self-consistency of the results. The present paper is thus organized as follow: in a first section the experimental setup is described. In a second section I discuss the statistical properties of the flow in the enstrophy cascade, and the dispersion of a blob of dye leading to the observation of the Batchelor regime. In the third section (which is the main section of the paper), the relative dispersion problem is presented in detail.

II. EXPERIMENTAL SETUP

There are several methods for imposing the dynamics of a flow to be two-dimensional. All of them use imposed external parameters such as aspect ratio, rotation, stratification or a magnetic field in the case of conducting fluids (see review of Tabeling\cite{Tabeling}). Two of these constraints are used to investigate two-dimensional turbulence in the laboratory: the aspect ratio and the density stratification.\cite{Boffetta}
A. The experimental flows

The experimental device is presented in a schematic way in Fig. 1. The flow is generated in a 16 cm×16 cm cell. The cell is filled by two layers of NaCl solution, placed in a stable configuration, i.e., the heavier underlying the lighter. Permanent magnets are located just below the bottom of the cell. Their magnetization axis is vertical and they produce a magnetic field, 0.3 T in maximum amplitude, which decays over a typical length of 3 mm. An electric current is driven through the cell from one side to the other. The interaction of this current with the magnetic field produces Laplace forces which drive the flow. Experiments carried out by Paret et al. on simple flows have shown that this device generates flows which are two-dimensional, i.e., which can be modeled by 2D Navier–Stokes equations. The main dissipation in this system is provided by the friction exerted by the bottom wall on the fluid. It can be parameterized by adding a linear term in the two-dimensional Navier–Stokes equations. This frictional term, if large enough, provides the infra-red energy sink preventing the condensation of energy in the lowest accessible mode. It can be varied by changing the total fluid depth but the range of allowed depths is not wide since it is clear that a large fluid depth would ruin the two-dimensionality assumption.

The flow is visualized by tiny latex particles placed at the free surface. During the experiment, the flow is recorded on a video tape using a CCD camera placed above the cell. The images are digitized and stored on a computer, at a frequency of 0.12 s between two following images. Standard particle imaging velocimetry (PIV) techniques are then used to compute the velocity fields at any time (for details of this method see Cardoso et al.). These fields are evaluated on 64×64 grids, corresponding to a 16×16 cm area. Finally, just by changing the magnet arrangement or the driving electric current, it is possible to investigate a wide class of flows.

In the experiments I present here, the magnets are arranged in such a way that the flow is initially concentrated around a prescribed wave number. The injection scale is about 10 cm for the enstrophy cascade (see Fig. 2 for the magnets arrangements). The total fluid depth is 6 mm. The imposed current, which, coupled to the magnetic field, defines the forcing, is a time series of impulses of constant amplitude, random sign, and random duration. The random-in-time forcing allows the imposition of an approximately zero net flow, which favors a randomness and tends to break the formation of coherent structures. The current is switched on at \( t = 0 \) and the flow is recorded for a typical time of 6 min for the analysis of the velocity fields. For experiments longer than 10 min, the stratification progressively disappears because of diffusion between the two layers, and consequently two-dimensionality is lost.

B. The tracer

The passive scalar is a mixture of fluorescein and water, of density \( \rho = 1030 \text{ gl}^{-1} \), and diffusivity \( \kappa = 10^{-6} \text{ cm}^2\text{s}^{-1} \). In most cases, the dye matches the upper layer density, and is vertically homogeneously spread across it, throughout the experiment. The concentration field is illuminated by ultraviolet light (365 nm). Figure 3 shows the relation between the concentration of the dye and the light code captured by IMAGE. Light codes are between 0 and 255, 0 corresponding to a white signal and 255 to a black one. The relation between the light code and concentration, shown on Fig. 3, is linear.

The tracer fields is visualized using a 512×512 CCD...
camera, encompassing a 15 cm × 15 cm region. The images are stored and further processed. One pixel corresponds then to the scale $r \sim 0.03$ cm. The typical length of diffusion after one minute (more than the typical duration of an experiment) is $l = \sqrt{D t} \approx 0.02$ cm. The “molecular” Péclet number $UL/\kappa$ (where $U$ is a typical velocity of the flow—roughly 1 cm/s) is on the order of $10^7$.

In order to check to what extent the results may depend on the characteristics of the fluorescent dye, the results are systematically compared with a “numerical” dye computed from the velocity fields.

C. The injection technique

To develop a Batchelor regime, the initial size of the dye blob has to be on the order of the large scale of the velocity field. To inject the dye, a blob of dye is first enclosed within a cylinder, 5 cm in diameter, in the upper layer at rest. The electrical current is then switched on, and when transient effects have vanished, the cylinder is delicately removed. The initial forcing is isotropic and in these experiments, the flow is statistically stationary, while the concentration field is in a freely decaying regime. However, it will be shown that there exists a range of time in which statistics are quasi-stationary.

In most cases a slightly lighter dye has been used, of density $\rho = 1002$ gl$^{-1}$, i.e., 3% lighter than the upper layer. Throughout the experiment, the tracer strings in the upper fluid layer. A phenomenon which tends to enhance diffusion has been identified, and therefore reduces the range of wave numbers in which the cascade develops. The phenomenon is visualized in Fig. 4 where it can be seen that the edges of the tracer filaments appear blurred. The origin of the phenomenon is a diffusion process taking place in the upper layer. The phenomenon is related to the presence of velocity gradients $\partial u/\partial z$, normal to the layer, which favors Taylor–Aris dispersion.

D. Determination of the Lagrangian trajectories

The method used to obtain Lagrangian trajectories is the same as the one used in Jullien et al.\textsuperscript{18} for the observation of Richardson law\textsuperscript{19} in the inverse energy cascade. The method consists as the following: the velocity fields, determined at all times by PIV technique, are used to compute trajectories $x(t)$ of simulated particles (which will be called simply “particles” later on); this is achieved by integrating the following Lagrangian equations of motion:

\[ \frac{d\mathbf{x}}{dt} = \mathbf{u}(t) \]

FIG. 5. Scatter of the pairs in the system.

FIG. 6. Energy spectra, in the enstrophy cascade, averaged over 200 realizations as a function of wave number $k$.

FIG. 7. Time evolution of a blob of fluoresceine of density $\rho = 1002$ gl$^{-1}$ in a 16 cm × 16 cm region, at times $t = 1, 12, 20, \text{ and } 40$ s.
The experimental conditions are such that an enstrophy cascade develops. The statistical study of the enstrophy cascade has been reported in Paret et al.\textsuperscript{8} and only the main characteristics will be underlined. The energy spectrum follows a $k^{-3/2}$ power law for scales ranging from 0.7 to 7 cm (see Fig. 6). This regime is found to be stationary, homogeneous, isotropic and non intermittent for the vorticity statistics. The estimation of the enstrophy transfer rate averaged over the inertial range gives $\beta \sim 0.25$ s$^{-1}$; the rate between the large scale $L$ and the dissipative scale $l_d = (v^3/\beta)^{1/6}$ is then around 100. The typical turnaround time, estimated from the enstrophy transfer rate, is $\tau_t \sim 1.6$ s.

### III. STATISTICAL PROPERTIES OF THE FLOW

#### A. The experimental flow: The enstrophy cascade

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#### B. The Batchelor regime

In this section, I briefly summarize the statistics of the dispersion of a blob of a dye freely decaying in the enstrophy cascade. This work has already been reported in Jullien et al.\textsuperscript{5} Only the main features will be showed in order to characterize more completely the experimental conditions and improve the self-consistency of the paper.

Figure 7 shows a typical evolution of a spot of fluoresceine 5 cm in diameter, released in the system at time $t = 0$. At early time, the blob is localized in the center of the cell, and is slightly distorted. It is further vigorously advected, strained and folded. These filaments can reach sizes comparable to the box size (15 cm) and spread over the whole cell. After a period of 30 s the concentration field is homogeneous. In this system, mixing seems homogeneous and there is no evidence of the presence of any long lived structure trapping the tracer throughout the duration of the experiment. We believe this is due to the particular forcing we use, which disrupts the coherent structures, a requirement for the development of an enstrophy cascade,\textsuperscript{9} and probably also for that of a Batchelor regime. Figure 8 shows the same fields in the case of the numerical dye, which shows the same characteristics.

A quasi-stationary domain has been determined by considering a range of time, lying between $8 \leq t \leq 18$ s, in which the total dissipation is constant (balance between production of tracer gradients by sequences of strains and destruction of gradients by diffusion).

The scalar variance spectra $E_\sigma(k)$, now averaged over the entire quasi-stationary domain, displays in Fig. 9 a range of wave numbers, between 1 and 7 cm$^{-1}$, where a $k^{-1}$ law fairly holds. Beyond $k = 7$ cm$^{-1}$, the spectrum drops (presumably because of the diffusion enhancement mentioned in Sec. II C).

A further analysis, detailed in Jullien et al.\textsuperscript{5} showed that the statistical properties of the passive tracer were in good qualitative agreement with theoretical analysis,\textsuperscript{3,4} i.e., exponential tails for both the PDFs of the concentration fluctuations and the PDFs of the concentration increments, and finally a logarithmic behavior for the structure function of order 2.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure8.png}
\caption{Time evolution of a blob obtained by integrating the experimental velocity field at times $t = 1, 12, 20, \text{and} 32$ s.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure9.png}
\caption{Variance scalar spectra with a pollutant of density $\rho = 1002$ gl$^{-1}$, the straight line has a $k^{-1}$ slope.}
\end{figure}
IV. RELATIVE DISPERSION IN THE ENSTROPHY CASCADE

A. Dispersion of a pair

Figure 10 shows the typical evolution of one pair of particles in the enstrophy cascade. The dispersion process is qualitatively different from the one in the inverse cascade, leading to Richardson law (see Jullien et al.18), and in which the dispersion process involved sequences of quiet periods and sudden bursts. The dispersion process, in the case considered here, does not seem to involve sudden bursts but appears to be a progressive process. The inset of Fig. 10 represents the time evolution of the pair separation \( r = \sqrt{(x_i-x_j)^2 + (y_i-y_j)^2} \) (subscript \( i \) referring to particle \( i \)) for the same particle pair, in a logarithmic-linear representation. It is noticeable that the particles separate exponentially from 0.6 to 4 cm, exploring inertial scales (almost one decade) in one turnaround time (i.e., 1.6 s). The separation process is a fast process in the sense that separation is more efficient than for the inverse cascade, in which a number of steps was necessary to reach a large scale separation.

B. The mean square displacement

Figure 11 shows the time evolution of the mean square displacement \( \langle R^2 \rangle \) obtained by averaging over \( 5 \times 10^4 \) pairs. One sees a vigorous increase of the separation, for which two regimes can be singled out: between 0 and 3 s an exponential law prevails, later, another regime seems to take place. At this stage, the system can be split in two domains: between 0 and 3 s, an exponential law applies, and beyond 3 s, an algebraic law seems to hold. One has to be cautious about such a decomposition: perhaps one could argue that between 0 and 1 s, the data are consistent with a \( \exp(t^{3/2}) \) law as proposed by Mirabel and Monin.9 In the present experiment however, the dynamical significance of the interval between 0 and 1 s is unclear, and, in such a context, it would be unsafe to claim that the \( \exp(t^{3/2}) \) law does apply in a domain where it is expected to hold. One should probably need broader scaling ranges to draw out a definite conclusion on that point. Also, in the present experiment, it seems wise to avoid generating too many (small) subdomains. One needs two subdomains at least to interpret the data, and for the rest of the paper I will assume this minimal number is also a maximum, an assumption consistent with the observations, within the experimental uncertainty.

The exponential regime can be made directly visible by plotting the local exponent \( \eta = d(\ln(\langle R^2 \rangle))/dt(\ln(t)) \) with time. The local exponent is represented in Fig. 12. The study is made for two different initial separations \( r_{01} = 0.03 \) cm and \( r_{02} = 0.06 \) cm. As for the observation of Richardson law in the inverse energy cascade,18 one may note that an initial separation much smaller than the small scale of the inertial range (at least one order of magnitude) is a requirement to observe a behavior different from a purely diffusive one.

In Fig. 12 there is a clear plateau located at \( \eta \approx 1 \) for times ranging from 1.5 to 3 s, i.e., during one turnaround time. The domain in which an exponential law is observed corresponds to a root mean square separation lying between 0.3 cm and 0.7 cm, which is rather small compared to the large scale 7 cm. The exponential law is then observed for times on the order of a turnaround time, but for scales smaller than 4 cm, exploring inertial scales almost one decade in one turnaround time.
smaller than the integral scale. This may not be so surprising, if we note that on such scales, the velocity increments are presumably no more a linear function of the separation.

For times greater than 3 s, an exponential behavior is no longer observed. In a search for a power law behavior the local exponent \( \alpha = d \langle \ln(R^2) \rangle/d \langle \ln(t) \rangle \) is plotted in Fig. 13. It shows a clear plateau \( \alpha = 3.6 \pm 1 \) for times ranging from 3 to 9 s. This range of time corresponds to scales ranging from 0.7 to 3.5 cm. A power law cannot be obtained by simple dimensional arguments à la Kolmogorov. Peskin,20 on the other hand, gave arguments that there is no requirement on the slope of the energy spectrum, in two dimensions, to obtain a power law for the dispersion process. In any case, the present measurements are consistent with the numerical simulations of Kowalski and Peskin.12

An other method to investigate the time evolution of the separation is the finite size Lyapunov exponents (FSLE), \( \lambda(\delta) \), introduced in the context of predictability problems, and generalized in the case of transport and mixing.13,14 Instead of looking at the mean square separation as a function of time, the FSLE method is based on the identification of the typical time \( \tau(\delta) \) characterizing the diffusive process at scale \( \delta \) through the exit time. Briefly: given a set of thresholds \( \delta^{(0)} = r^{\alpha} \delta^{(0)} \), one can measure the time \( T_i(\delta^{(0)}) \) it takes for the separation \( R_i(t) \), of pair \( i \), to grow from \( \delta^{(0)} \) to \( \delta^{(1)} = r \delta^{(0)} \), and so on for \( T_i(\delta^{(1)}), T_i(\delta^{(2)}), \ldots \) up to the largest considered scale. The \( r \) factor may be any value \( >1 \), and is equal to \( \sqrt{2} \) in the present paper. For a set of \( N \) particle pairs, the average growing time \( \tau(\delta) \) at scale \( \delta \) is defined as

\[
\tau(\delta) = (T(\delta))_e = \frac{1}{N} \sum_{i=1}^{N} T_i(\delta).
\]

The finite size lagrangian Lyapunov exponent is then defined as

\[
\lambda(\delta) = \frac{\ln r}{\tau(\delta)},
\]

which quantifies the average rate of separation between two particles at a distance \( \delta \). In terms of the finite size Lyapunov exponent, the following different regimes are expected, depending on the dispersion process:

1. \( \lambda(\delta) \sim \delta^{-2\alpha} \) if \( \langle R^2(t) \rangle \sim t^a \) (\( \alpha = 1 \) for purely diffusive process and \( \alpha = 3 \) in the inertial range of both direct cascade in 3D and inverse cascade in 2D);
2. \( \lambda(\delta) = \lambda \) if the flow is smooth (expected in the direct enstrophy cascade if there is no logarithmic corrections);
3. \( \lambda(\delta) \sim (\ln(\delta))^{1/2n} \) if \( \langle R^2(t) \rangle \sim R(0) \exp(c^n t) \) (\( c^n \) is a constant and \( \alpha = 3/2 \) if logarithmic corrections are expected).

Figure 14 represents \( \lambda \) as a function of \( \delta \) for two different initial separations \( r_{01} = 0.03 \) cm and \( r_{02} = 0.06 \) cm. As for the previous analysis, an exponential behavior is observed for scales ranging from 0.1 cm to 1 cm and then a power law regime is observed. In the exponential regime, \( \lambda = 5.65 \pm 0.22 \) for \( r_{01} \) and \( \lambda = 5.39 \pm 0.06 \) for \( r_{02} \). In the power law regime, \( -2/\alpha = 0.43 \pm 0.15 \) for \( r_{01} \) and \( -2/\alpha = 0.4 \pm 0.075 \) for \( r_{02} \), i.e., \( \alpha \sim 5 \) for both \( r_{01} \) and \( r_{02} \). Even with this method, which is known as a powerful tool for unambiguous results,22 the \( \exp(c^n t^{3/2}) \) is not observed. These findings are consistent with those of the beginning of the section: the two regimes are recovered, with the same characteristics. It is interesting to note that using the FSLE method, one gets a wider range of existence for the exponential regime, which now appears quite neatly. This advantage is obtained at the expense of the algebraic regime, which is less difficult to detect on the plot of Fig. 13. As a whole, one may say that, by either method (fixed time or exit time) one get similar results, which reinforces the self-consistency of the results.

C. PDFs of the pair separations

1. In the exponential regime

I now turn to the PDFs of the pair separations. They have been obtained for an initial separation \( r_0 = 0.12 \) cm, and for 25 000 pairs. They are represented in Fig. 15(a) for 1 to 3.5 s every half a second. As time grows they develop large tails. Rescaled so as the variance be unity [see Fig. 15(b)].
the tails of the PDFs, defined as events \( dr/\sigma > 1 \), collapse on a single curve which can be fitted by a stretched exponential
\[ \exp(-2.2 \times (dr/\sigma)^{0.55 \pm 0.05}), \]
revealing that the process is self-similar in time.

2. In the power law regime

The PDFs of the pair separations are represented in Fig. 16(a) for 4 to 10 s every second, and only for inertial scales. They develop exponential tails. When rescaled so as the variance be unity, considering events \( dr/\sigma > 1 \), and removing events larger than inertial scales, i.e., \( dr > 8 \) cm, they collapse on a single curve up to 8 s, as seen in Fig. 16(b). The dispersion process is then self-similar in time in this regime. The shape of the tails are different from those of Fig. 15, revealing a striking difference between the two temporal regimes. To my knowledge, there is no prediction for the shape of these distributions.

D. Lagrangian correlations

1. In the exponential regime

It is instructive to characterize separation temporal correlations. The correlation functions of pair separations, i.e., the quantity:
\[ R(t, \tau) = \langle r(t) r(t, \tau) \rangle, \quad \text{with} \ -t \leq \tau \leq 0, \]
is shown in Fig. 17, rescaled by \( R(t,0) \) for six different times between 1 and 3.4 s. \( \tau \) is negative in order to investigate the way the pairs are correlated with their own history. Without any rescaling, the curves collapse on a single curve; this defines a constant correlation time. From Fig. 17, this time is estimated to be \( \tau_c \sim 2.5 \) s, which is of the same order of

FIG. 15. (a) PDFs of the pair separations at times \( t = 1 \) to 3.5 s at each half a second, from left to right. (b) Rescaled PDFs so as the variance be unity. Straight line is a stretched exponential fit \( \exp(-2.2 \times (dr/\sigma)^{0.55 \pm 0.05}) \).

FIG. 16. (a) PDFs of the pair separations at times \( t = 4 \) to 10 s at each second, from left to right. (b) Rescaled PDFs so as the variance be unity. Straight line is an exponential fit.

FIG. 17. Lagrangian separation correlation factor \( R(t, \tau) \) for \( t = 1, 1.6, 2, 2.6, 3, \) and 3.4 s.
magnitude of one turnaround time \( \tau_t = \beta^{-1/3} \sim 1.6 \) s. Consequently, in the exponential regime, a single correlation time holds, one may define as being

\[ \tau_c \sim C \times \beta^{-1/3}, \]

where \( C \) is a constant estimated to be 1.7 in our experiment. Note that I have not checked whether such a relation holds in general, since \( b \) could not be sensitively varied in the experiment.

2. In the power law regime

At intermediate times, i.e., in the range of time in which a power law regime is observed, the correlation function of separation is shown in Fig. 18, rescaled by \( R(t,0) \), as a function of \( \tau/t \) at times \( t = 6, 8, \) and 10 s. As time grows, the time beyond which the particles decorrelate raises up. Similarly as for the PDFs, the curves collapse on a single curve. This suggests the general form for the correlation function:

\[ R(t, \tau)/R(t,0) = f(\tau/t), \]

where \( f(\tau/t) \) is a dimensionless function. The physical Lagrangian correlation \( \tau_c \), estimated from Fig. 18, is

\[ \tau_c \sim 0.60 t, \]

which underlines the persistence of correlations.

E. PDFs of Lagrangian separation velocities

In order to document the statistics, I have studied the Lagrangian velocity of separation, defined has the derivative of separation increments between two times, evaluated by 0.2 s. The Lagrangian separation velocity is then defined as

\[ \delta v(t) = \frac{d \delta r(t)}{dt} = v^L_2(t) - v^L_1(t), \]

where \( d \delta r(t) \) is the separation increment during \( dt \) (elapsed time between two successive fields), \( v^L_2(t) \) the Lagrangian velocity of particle 2 and \( v^L_1(t) \) the one of particle 1.

Curves in Fig. 19(a) represent the PDFs of the Lagrangian separation velocities, for \( 25 \times 10^3 \) pairs, at different times between 3 and 11 s after the particles are released in the system. They have been rescaled by the root mean square calculated at each time [Fig. 19(b)]. This distributions are nonsymmetric. This asymmetry can be understood since, on the average, particles separate (it is then expected that there are more positive events than negative ones). When rescaled so as the variance be unity [see Fig. 19(b)], the curves collapse on a single curve up to \( 8 \times (\delta v^2)^{1/2} \) (it is worth noting that the rare events are in the range of \( 25 \times (\delta v^2)^{1/2} \)). Once again, temporal self-similarity is obtained consistently with the previous sections.

V. DISCUSSION: CONCLUSION

In this work I have examined the dispersion of pairs of particle in the direct enstrophy cascade, in a two-dimensional experimental turbulent flow, where energy is injected at large scales.

The study of the single pair dispersion highlights a fast separation process since the pair separation explore one decade of scales in one turnaround time. The temporal evolution of the mean square separation displays an exponential behavior during one turnaround time. The \( \exp(A(t/\tau_t)^{1/2}) \) (where \( A \) is a constant) behavior, expected when logarithmic corrections are taken into account, is not observed. The exponential behavior is observed at early times between one to
two turnaround time and for scales ranging from 0.1 to 1 cm. At later time, a power law behavior is observed. At the moment, there is no theoretical expectation for this behavior.

This study raises a few issues. First of all, it seems surprising that the consequences of the existence of logarithmic corrections are not observed. Instead, Lin’s law is obtained. This result remains to reconcile with theory. These experimental observations do not give clues for the physical understanding of the lack for logarithmic corrections. Furthermore, the range of time within which Lin’s law is obtained is on the order of the large time scale, consistently with theoretical expectations. Nevertheless, the corresponding range of spatial scales, within which Lin’s law applies, can be found surprisingly small: it extends up to less than one tenth of the scale at which energy is injected. This observation, consistent with previous works, is puzzling. It is not clear to understand why the domain is so small. One possibility (mentioned earlier) is the nonlinear behavior of the velocity increments with the scale, at “large” increments. In any case, Lin’s law is rather difficult to observe, and its relevance may appear to control a comfortable range of scales. Its origin is at the moment unclear, although some attempts to justify it have been made in the past. There is no hint about the universality or nonuniversality of the exponent found here, and at the moment, available data are too scarce to formulate any conclusion. Finally, it may be strange that the situation, concerning the pair dispersion, is more complicated than its Eulerian counterpart (which could be compared with the theory). It seems that much work remains to be done to fully understand the phenomenon I analyze in the present paper, despite Batchelor regime is reputed to be a solved problem.

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